

<p><b>Claim 1:</b> Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.</p>	
<p>Content Domain: <b>Functions</b></p>	
<p><b>Target N [m]:</b> Build a function that models a relationship between two quantities. (DOK 2)</p> <p>Tasks for this target will require students to write a function (recursive or explicit, as well as translate between the two forms) to describe a relationship between two quantities.</p>	
<p>Standards:  F-BF.A, F-BF.A.1 F-BF.A.1a, F-BF.A.2</p>	<p><b>F.BF.A Build a function that describes a relationship between two quantities.</b></p> <p><b>F-BF.A.1</b> Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p><b>F-BF.A.2</b> Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.</p>
<p>Related Below Grade and Above Grade Standards for Purposes of Planning for Vertical Scaling:  8.F.A, 8.F.A.1  8.F.B, 8.F.B.4, 8.F.B.5</p>	<p><b>Related Grade 8 Standards</b></p> <p><b>8.F.A Define, evaluate, and compare functions.</b></p> <p><b>8.F.A.1</b> Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.</p> <p><b>8.F.B Use functions to model relationships between quantities.</b></p> <p><b>8.F.B.4</b> Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two <math>(x, y)</math> values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p> <p><b>8.F.B.5</b> Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</p>
<p>DOK Level(s):</p>	<p>2</p>

<b>Achievement LEVEL Descriptors:</b>	
<p><b>RANGE Achievement Level Descriptors (Range ALD)</b> Target N: Build a function that models a relationship between two quantities.</p>	<p><b>Level 1</b> Students should be able to identify an explicit or a recursive function and determine the steps for calculation from a context requiring up to two steps. They should be able to add and subtract two linear functions.</p>
	<p><b>Level 2</b> Students should be able to build an explicit or a recursive function to describe or model a relationship between two quantities and determine the steps for calculation from a context. They should be able to add, subtract, and multiply linear and quadratic functions.</p>
	<p><b>Level 3</b> Students should be able to translate between explicit and recursive forms of a function. They should be able to add, subtract, multiply, and divide functions.</p>
	<p><b>Level 4</b> Students should be able to determine when it is appropriate to combine functions using arithmetic operations in context.</p>
<p>Evidence Required:</p>	<p>1. The student writes explicit or recursive functions to describe relationships between two quantities from a context.</p> <p>2. The student translates between explicit formulas and recursively defined functions.</p> <p>3. The student understands a function as a model of the relationship between two quantities.</p>
<p>Allowable Response Types:</p>	<p>Multiple Choice, single correct response; Equation/Numeric; Matching Table; Fill-in Table</p>
<p>Allowable Stimulus Materials:</p>	<p>The student is presented with a contextual situation familiar to 16–17 year olds where a function can describe a relationship between two quantities. Contextual situations will be introduced with simple subject-verb-object sentences, avoiding long noun phrases, multiple prepositional phrases, and unfamiliar technical vocabulary. Specific stimuli include: explicit functions, recursive functions, written descriptions of functional relationships between two quantities, a sequence of numbers, a table representing a sequence of numbers and corresponding term numbers, a sequence in which the first four numbers are given or any four consecutive terms excluding <math>a_1</math>.</p>
<p>Construct-Relevant Vocabulary:</p>	<p>Function(s), quantity, quantities, explicit, recursive, arithmetic sequence, geometric sequence, input, output, ordered pairs</p>
<p>Allowable Tools:</p>	<p>calculator</p>
<p>Target-Specific Attributes:</p>	<p>When translating between an explicit function and a recursive function, functions are limited to arithmetic and geometric relations.</p>
<p>Non-Targeted Constructs:</p>	<p>functions, function notation, sequences, sequence notation, <math>n</math>th term, common difference, common ratio</p>

<p>Accessibility Guidance:</p>	<p>Item writers should consider the following Language and Visual Element/Design guidelines<sup>1</sup> when developing items.</p> <p>Language Key Considerations:</p> <ul style="list-style-type: none"> <li>• Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context</li> <li>• Avoid sentences with multiple clauses</li> <li>• Use vocabulary that is at or below grade level</li> <li>• Avoid ambiguous or obscure words, idioms, jargon, unusual names and references</li> </ul> <p>Visual Elements/Design Key Considerations:</p> <ul style="list-style-type: none"> <li>• Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context</li> <li>• Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary</li> <li>• Avoid crowding of details and graphics</li> </ul> <p>Items are selected for a student’s test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.<sup>2</sup></p>
<p>Development Note:</p>	<p>The modeling described in Standard F-FB.A.2 should be measured in Claim 4.</p>

<sup>1</sup> For more information, refer to the General Accessibility Guidelines at: <http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf>

<sup>2</sup> For more information about student accessibility resources and policies, refer to [http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced\\_Guidelines.pdf](http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf)

<p><b>Task Model 1</b></p> <p><b>Response Type:</b> Equation/Numeric</p> <p><b>DOK Level 2</b></p> <p><b>F-BF.A.1</b> Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p><b>Evidence Required:</b> 1. The student writes explicit or recursive functions to describe relationships between two quantities from a context.</p> <p><b>Tools:</b> Calculator</p>	<p><b>Prompt Features:</b> The student is prompted to enter a function that represents a relationship between two quantities by determining an explicit function that represents a context.</p> <p><b>Stimulus Guidelines:</b> The student is presented with a contextual situation that describes a relationship between two quantities that can be modeled by a function.</p> <ul style="list-style-type: none"> <li>• Functions can be linear, quadratic, exponential, or rational.</li> <li>• Difficulty level can be altered by varying the type of function and context.</li> </ul> <p><b>TM1a</b> <b>Stimulus:</b> The student is presented with a contextual situation.</p> <p><b>Example Stem 1:</b> Maria is making a rectangular garden. The length of the garden is 2 yards greater than its width, <math>w</math>, in yards.</p> <p>Enter the function, <math>f(w)</math>, that describes the area, in square yards, of Maria’s garden as a function of the width, <math>w</math>.</p> <p><b>Example Stem 2:</b> Barb traveled 300 miles during the first 5 hours of her trip. Barb then traveled at a constant speed of 50 miles per hour for the remainder of the trip.</p> <p>Enter the function, <math>f(t)</math>, that describes the average speed during the entire trip as a function of time, <math>t</math>, in hours, Barb traveled after her first 300 miles.</p> <p><b>Example Stem 3:</b> A washing machine was purchased for \$256. Each year the value is <math>\frac{1}{4}</math> of its value the previous year.</p> <p>Enter the function, <math>f(t)</math>, that describes the value of the washing machine, in dollars, as a function of time in years, <math>t</math>, after the initial purchase.</p> <p><b>Rubric:</b> (1 point) The student correctly enters the function describing the relationship between two quantities in the given contextual situation (e.g., <math>f(w) = w(w + 2)</math>; <math>f(h) = \frac{300+50h}{5+h}</math>; <math>f(t) = \\$256(0.75)^t</math>).</p> <p><b>Response Type:</b> Equation/Numeric</p>
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<p><b>Task Model 1</b></p> <p><b>Response Type:</b> Equation/Numeric</p> <p><b>DOK Level 2</b></p> <p><b>F-BF.A.1</b> Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p><b>Evidence Required:</b> 1. The student writes explicit or recursive functions to describe relationships between two quantities from a context.</p> <p><b>Tools:</b> Calculator</p> <p><b>Version 3 Update:</b> Retired TM1b Example Stem 2.</p>	<p><b>Prompt Features:</b> The student is prompted to enter a function that describes a relationship between two quantities by determining a recursive rule that represents a context.</p> <p><b>Stimulus Guidelines:</b> The student is presented with a contextual situation that describes a relationship between two quantities that can be modeled by a function.</p> <ul style="list-style-type: none"> <li>Function types can include linear or exponential.</li> <li>Difficulty level can be altered by varying the type of function and context.</li> </ul> <p><b>TM1b</b> <b>Stimulus:</b> The student is presented with a contextual situation.</p> <p><b>Example Stem:</b> A researcher studies the growth of a fruit fly population. The researcher counts the number of fruit flies at noon each day. The results are in the table.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Day</th> <th>Number of Fruit Flies</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>4</td> </tr> <tr> <td>1</td> <td>8</td> </tr> <tr> <td>2</td> <td>16</td> </tr> <tr> <td>3</td> <td>32</td> </tr> </tbody> </table> <ul style="list-style-type: none"> <li><math>V(n)</math> = Total number of fruit flies after <math>n</math> days</li> <li><math>V(0) = 4</math></li> </ul> <p>Enter the function for <math>n \geq 1</math>, which describes the number of fruit flies, <math>V(n)</math>, at noon on the <math>n^{\text{th}}</math> day in terms of the number of fruit flies at noon on the previous day, <math>V(n - 1)</math>.</p> <p><b>Rubric:</b> (1 point) Student correctly enters the function describing the relationship between two quantities in the given contextual situation [e.g., <math>V(n) = 2V(n - 1)</math>].</p> <p><b>Response Type:</b> Equation/Numeric</p>	Day	Number of Fruit Flies	0	4	1	8	2	16	3	32
Day	Number of Fruit Flies										
0	4										
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<p><b>Task Model 1</b></p> <p><b>Response Type:</b> Equation/Numeric</p> <p><b>DOK Level 2</b></p> <p><b>F-BF.A.1</b> Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p><b>Evidence Required:</b> 1. The student writes explicit or recursive functions to describe relationships between two quantities from a context.</p> <p><b>Tools:</b> Calculator</p>	<p><b>Prompt Features:</b> The student is prompted to model a given contextual situation as a sequence using either a recursively defined function or an explicitly defined function.</p> <p><b>Stimulus Guidelines:</b> The student is presented with a contextual description of two quantities.</p> <ul style="list-style-type: none"> <li>• The context can be modeled by:             <ul style="list-style-type: none"> <li>○ an arithmetic sequence</li> <li>○ a geometric sequence</li> <li>○ Items may also draw upon knowledge contained in F-IF.A.3.</li> </ul> </li> <li>• Difficulty level can be altered by varying the complexity of function and context.</li> </ul> <p><b>TM1c</b> <b>Stimulus:</b> The student is presented with a contextual situation.</p> <p><b>Example Stem 1:</b> The first row in a theater has 8 seats, the second row has 11 seats, the third row has 14 seats and the fourth row has 17 seats. The pattern of increasing each successive row by 3 seats continues throughout the theater.</p> <ul style="list-style-type: none"> <li>• <math>f(r)</math> = the number of seats in row <math>r</math>.</li> <li>• <math>f(1) = 8</math></li> </ul> <p>Enter an equation, for <math>r \geq 2</math>, which describes the number of seats, <math>f(r)</math>, in the <math>r</math>th row in terms of the number of seats in the <math>(r - 1)</math>th row, <math>f(r - 1)</math>.</p> <p><b>Example Stem 2:</b> The 13th row in a theater has 41 seats, the 12th row has 38 seats, the 11th row has 35 seats and the 10th row has 32 seats. The pattern of decreasing each successive row by 3 seats continues from the 13th row to the 1st row.</p> <ul style="list-style-type: none"> <li>• <math>f(r)</math> = the number of seats in row <math>r</math>.</li> <li>• <math>f(1) = 5</math></li> </ul> <p>Enter an equation, for <math>r \geq 2</math>, that describes the number of seats, <math>f(r)</math>, in the <math>r</math>th row in terms of the number of seats in the <math>(r - 1)</math>th row, <math>f(r - 1)</math>. Assume that the pattern described applies to all rows.</p> <p><b>Rubric:</b> (1 point) Student correctly represents the sequence with a recursively defined function [e.g., <math>f(r) = f(r - 1) + 3</math>; <math>f(r) = f(r - 1) + 3</math>].</p> <p><b>Response Type:</b> Equation/Numeric</p>
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<p><b>Task Model 2</b></p> <p><b>Response Type:</b> <b>Multiple Choice, single correct response</b></p> <p><b>DOK Level 2</b></p> <p><b>F-BF.A.2</b> Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.</p> <p><b>Evidence Required:</b> 2. The student translates between recursive functions and explicit functions.</p> <p><b>Tools:</b> Calculator</p>	<p><b>Prompt Features:</b> The student is prompted to select a recursive or explicit function that is equivalent to a given function.</p> <p><b>Stimulus Guidelines:</b></p> <ul style="list-style-type: none"> <li>• Sequences can be either arithmetic or geometric in a given item.</li> <li>• Domain should only include integers.</li> <li>• <math>a_1</math> needs to be less than or equal to <math>\pm 20</math>.</li> <li>• Difference between numbers in arithmetic sequence should be less than or equal to five.</li> <li>• Difficulty level can be altered by varying the type of function and context.</li> </ul> <p><b>TM2a</b> <b>Stimulus:</b> The student is presented with an explicit or recursively defined function.</p> <p><b>Example Stem 1:</b> Consider this function in explicit form.</p> $f(n) = 3n - 4; n \geq 1$ <p>Select the equivalent recursively defined function.</p> <p>A. <math>f(1) = -1</math> <math>f(n) = f(n - 1) + 3; n \geq 2</math></p> <p>B. <math>f(1) = -1</math> <math>f(n) = 3f(n - 1); n \geq 2</math></p> <p>C. <math>f(0) = -4</math> <math>f(n) = 3f(n - 1); n \geq 2</math></p> <p>D. <math>f(0) = -4</math> <math>f(n) = f(n - 1) + 3; n \geq 2</math></p> <p><b>Example Stem 2:</b> Consider this function in recursive form.</p> $f(1) = -3$ $f(n) = 3f(n - 1); n \geq 2$ <p>Select the equivalent explicit function for <math>n \geq 1</math>.</p> <p>A. <math>f(n) = -3(n)</math> B. <math>f(n) = -3(n - 1)</math> C. <math>f(n) = -3(3)^n</math> D. <math>f(n) = -3(3)^{(n-1)}</math></p> <p><b>Rubric:</b> (1 Point) Student selects the correct choice (e.g., A; D).</p> <p><b>Response Type:</b> Multiple Choice, single correct response</p>
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<p><b>Task Model 2</b></p> <p><b>Response Type:</b> <b>Matching Tables</b></p> <p><b>DOK Level 2</b></p> <p><b>F-BF.A.2</b> Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.</p> <p><b>Evidence Required:</b> 2. The student translates between recursive functions and explicit functions.</p> <p><b>Tools:</b> Calculator</p>	<p><b>Prompt Features:</b> The student is prompted to match explicitly defined functions with their equivalent recursive form.</p> <p><b>Stimulus Guidelines:</b></p> <ul style="list-style-type: none"> <li>All explicit functions will have an equivalent recursive function.</li> <li>Sequences can be either arithmetic or geometric in a given item.</li> <li>Domain should only include integers, excluding rational numbers.</li> <li><math>a_1</math> needs to be less than or equal to <math>\pm 20</math>.</li> <li>Difference between numbers in arithmetic sequence should be less than or equal to five.</li> <li>Difficulty level can be altered by varying the type and complexity of function.</li> </ul> <p><b>TM2b</b></p> <p><b>Stimulus:</b> The student is presented with explicit and recursive functions.</p> <p><b>Example Stem:</b> The functions in the table are defined for integers <math>n \geq 1</math>. Match each recursively defined function with the equivalent explicit form.</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="padding: 5px;">Functions</th> <th style="padding: 5px;"><math>f(n) = 3(10)^{(n-1)};</math> <math>n \geq 1</math></th> <th style="padding: 5px;"><math>f(n) = 3n + 7;</math> <math>n \geq 1</math></th> <th style="padding: 5px;"><math>f(n) = 10(3)^{(n-1)};</math> <math>n \geq 1</math></th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;"><math>f(1) = 10</math> <math>f(n) = 3f(n - 1);</math> <math>n \geq 2</math></td> <td style="width: 50px; height: 40px;"></td> <td style="width: 50px; height: 40px;"></td> <td style="width: 50px; height: 40px;"></td> </tr> <tr> <td style="padding: 5px;"><math>f(1) = 3</math> <math>f(n) = 10f(n - 1);</math> <math>n \geq 2</math></td> <td style="width: 50px; height: 40px;"></td> <td style="width: 50px; height: 40px;"></td> <td style="width: 50px; height: 40px;"></td> </tr> <tr> <td style="padding: 5px;"><math>f(1) = 10</math> <math>f(n) = f(n - 1) + 3;</math> <math>n \geq 2</math></td> <td style="width: 50px; height: 40px;"></td> <td style="width: 50px; height: 40px;"></td> <td style="width: 50px; height: 40px;"></td> </tr> </tbody> </table> <p>Click the appropriate box that matches the recursive form in the first column with its equivalent explicit form in the top row.</p> <p><b>Interaction:</b> The student is presented with three explicit functions in the first row and three recursive functions in the first column. The student selects the cell in the table that matches the functions.</p> <p><b>Rubric:</b> (1 point) Student correctly matches all functions (e.g., see below).</p>	Functions	$f(n) = 3(10)^{(n-1)};$ $n \geq 1$	$f(n) = 3n + 7;$ $n \geq 1$	$f(n) = 10(3)^{(n-1)};$ $n \geq 1$	$f(1) = 10$ $f(n) = 3f(n - 1);$ $n \geq 2$				$f(1) = 3$ $f(n) = 10f(n - 1);$ $n \geq 2$				$f(1) = 10$ $f(n) = f(n - 1) + 3;$ $n \geq 2$			
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**Response Type:** Matching Tables

<p><b>Task Model 3</b></p> <p><b>Response Type:</b> <b>Fill-in Table</b></p> <p><b>DOK Level 2</b></p> <p><b>F-BF.A.1</b> Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p><b>Evidence Required:</b> 3. The student understands a function as a model of the relationship between two quantities.</p> <p><b>Tools:</b> Calculator</p> <p><b>Version 3 Update:</b> Retired TM3a</p>	<p><b>Prompt Features:</b> The student is prompted to model a given contextual situation as a sequence given a recursively defined function.</p> <p><b>Stimulus Guidelines:</b></p> <ul style="list-style-type: none"> <li>• The student is presented with a contextual description of two quantities that can be modeled by:             <ul style="list-style-type: none"> <li>○ an arithmetic sequence</li> <li>○ a geometric sequence</li> <li>○ Items may also draw upon knowledge contained in F-IF.A.3.</li> </ul> </li> <li>• Difficulty level can be altered by varying the type of function and context.</li> </ul> <p><b>TM3b</b> <b>Stimulus:</b> The student is presented with a contextual situation.</p> <p><b>Example Stem:</b> A theater needs to place seats in rows. The function, <math>f(r)</math>, as shown below, can be used to determine the number of seats in each row, where <math>r</math> is the row number.</p> $f(1) = 8$ $f(r) = f(r - 1) + 3$ <p>Use the function to complete the table indicating the number of seats in each of the first four rows of the theater.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="padding: 5px;">Row number</th> <th style="padding: 5px;">Number of Seats</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">Row 1</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px;">Row 2</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px;">Row 3</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px;">Row 4</td> <td style="padding: 5px;"></td> </tr> </tbody> </table> <p><b>Rubric:</b> (1 point) Student correctly enters the sequence from the recursive form into the table (e.g., see below).</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="padding: 5px;">Row number</th> <th style="padding: 5px;">Number of Seats</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">Row 1</td> <td style="padding: 5px;">8</td> </tr> <tr> <td style="padding: 5px;">Row 2</td> <td style="padding: 5px;">11</td> </tr> <tr> <td style="padding: 5px;">Row 3</td> <td style="padding: 5px;">14</td> </tr> <tr> <td style="padding: 5px;">Row 4</td> <td style="padding: 5px;">17</td> </tr> </tbody> </table> <p><b>Response Type:</b> Fill-in Table</p>	Row number	Number of Seats	Row 1		Row 2		Row 3		Row 4		Row number	Number of Seats	Row 1	8	Row 2	11	Row 3	14	Row 4	17
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